

Cardinal invariants and the generalized Baire spaces

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Der Wissenschaftsfonds.

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- Classic cardinal invariants and their generalizations
 - A word on Cichón's diagram
 - The generalized ultrafilter number

- The independence number

Section 1

Classic cardinal invariants and their generalizations

K R j C q j C

+ a0CN I CNq aC Njc a3 cCLUIw j.@~38RLa l@Cj,.,@aQ CaNC Rm
jan3 8R.a\$3,RL3 8 lc3YYY

N0a3 c #l+dRcL\$CN jRaC I, a0CN I, @ a ,j3aC.cljCSz R

+l ccC, I, a0CN I CNq aC Njc R8@@@3#\$C3a3 8Uj3B lcCq3I
N0 nN03acjRR0Y BN 8 ,j. Cj Cc URccC\$13 jR 0Ca3,jl,w \$
jR N a\$Cja aw nN,RnNjY\$13, a0CN I

i @ 3 < 3 N 3 a | C y 3 0 # C

H3 † \$3 N n N, R n N j \$ I 3 a 3 < n l a , a 0 Ę N t t y j i @ c 3 w 3 C N 3 a I C
C a 3 c U , 3 C c E n c j j @ 3 c t 3 j 3 N 0 8 R n 3 , g G R N @ j @ 3 j R U R I R < w
j @ 3 c 3 j c R 8 j @ < 3 ' s R a \ - Ę Ę C g " 8 R " a n t Ę Y

/ 3 N R j r / Ę j R \$ 3 j @ 3 , R I I 3 , j C R N R 8 N R s @ 3 d 3 Ę 0 3 0 j c @ a 3 \$ 0 3
j R j @ C c j R U R I R < w . a 3 , f i t Ę @ N R s @ j 3 a 3 C 3 8 N 8 3 a ' 3 q 3 t Ę w
j @ 3 a 3 3 # C g j : c n , @ j @ # j > Ę ö Y â

i@3N Cj s3 03~N3 j@3 k8IN3 α 10jyc30Nj R \$BAnNCRNc R8 3I3
CMr/Ē N0 03NRjβjR \$3 j@A3C 03 Itj@L3 <3a c3jc 03j3aLCM
tAC03 IL3 Nc NC03 Ij@ jCN 00CjCRN Cc ,IR 030 nN0

Bj Cc s3II GNRsN j@ jj@3 # Ca3 , j3<Raw j@3Ra3L , N \$
@RI0c j@ jj@3 CNjAdc N,yCRUN R 03Nc3 c3jc Cc RU3N V7a
FnICGRq. lzS:WY

i @ 3 \$ 3 < C N I

bCN,3 SOO9. sCj@+j@3CNUI3@Nq aC Njc \$Rq38ja@R1,FnLJA
LCN<c N0 b@3l @. j@3 cjn0w R8 j@3 CNq aC Njc ccR,C
CNj3a ,jCRNc @ c \$33N 03q3IRUCN<Y

Bj Cc IcR CLURaj NjjR L3NjCRN j@ jj@3 cjn0w R8 j@
UaR ,@30 8aRL j@3 URCNj R8 qC3s R8 /3c,aCUjCq3 b3.
?wjCN3N. FnICGRq. KRa3NR. KRjjR `RcW N0 iRURIR<v

bRL3 , a0CN I CN

/3 ~ NCjCRN

B8 a3 8nN, jCRN s3CN w j@ j- . C8 j@ 3a3 3k CdjcnN@ j@ j@
8Ra I I k. | | Y BN j@ Cc , c3. s33q 3Nj@ jlw 0RLCN

/3 ~ NCjCRN

H3j \$3 8 LCIw R8 8nN, tjCRN c 8aRL
Ä • C0RLCN .jC 8Ra n t f. j@ 3a3 3u Ccj c• dN , @ j@ j- Y
Ä • C 0N \$R n N 00380 8Ra n t f. j@ 3a3 3u Ccj c• dN , @ j@ *j- Y

i @ 3 n N \$ R n N 0 C N < N 0 0 R L C N

/3 ~ NCjCRN

Ä i @ 3 n N \$ R n N 0 C N < N n L \$ 3 a -

— t L C N] •] C C • c N n N \$ R n N 0 3 0 8 L C I w t R 8 8 n N , j C

Ä i @ 3 0 R L C N j C N < N n L \$ 3 a -

™ t L C N] •] C C • c 0 R L C N j C N < 8 L C I w t R 8 8 n N , j C

+ a0CN I CNq aC Njc ccR,C

H3è\$3 tAC03 I V,IRc30a0y30 nNctRncW RN

/3~NCjCRN

Ä i@3 00CjCqCjw NnL\$3a-

00 è LCW]é]C é f 0è0É é p è ^

Ä i@3 ,Rq3aCN< NnL\$3a-

,Rqè LCW]é]C é f 0è0É é t È ^

/3~NCjCRN

Ä i@3 ,R~N ICjw NnL\$3a-

,R8è LCN]é]C é f è 0 8Ra nIè j@3a3 Cc
ÿ n ésCj@f ÿ ^

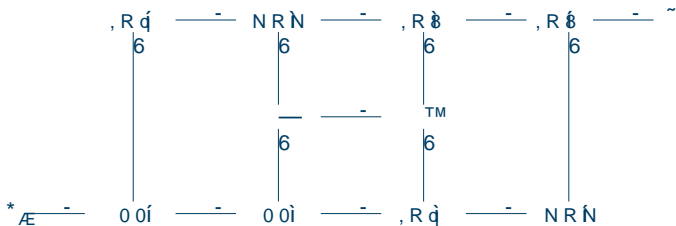
Ä i@3 nNC8RaLCjw NnL\$3a-

NRè LCN]]C b NO p è ^

+ C , @ 1/2 N c 0 C

+C, @ 1/2 N c 0 C <a L cnLL aCy87+ja@ B jCaRMc@ G Uc \$3js33M
0CN I CNq aC Njc a|3Acj33 jR R@ 1 3 <3a N0 NnI c3jc VsC
cj N0 a0 UaR0n,j L3 cna3W RN j@3 ,l ccC, l# Ca3 cU , 3

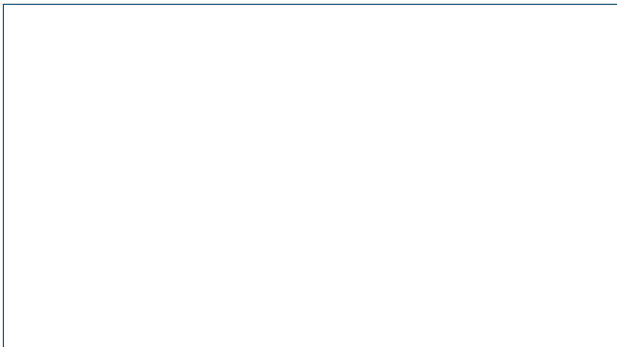
+ C , @ R « c / C < a L R N j @ 3 ' #



7 C < n a 3 S - + C , @ ½ N c 0 C < a L

? cc3 c 0C <

▶ :R jR CN03U3N03N,



r @ w , a 0 C N I C N q a C N j c R B j

i @ 3 a 3 a 3 c R L 3 a 3 L a G \$ I 3 0 C { 3 a 3 N , 3 c \$ 3 j s 3 3 N j @ 3 , R n N
, c 3 c j @ j L G 3 j @ C c c j n 0 w C N j 3 a 3 c j C N < N 0 U a 3 c 3 N j
a 3 c 3 a , @ Y ? 3 a 3 c R L 3 3 u L U I 3 c -

Ä 2 u U 3 , j 3 0 \$ R n N 0 C , I l w . , I c c C , I C N q a C N j c j G 3 q
< * Æ ~ > ? R s 3 q 3 a . C N j @ 3 n N , R n N j \$ I 3 8 R a C N c j N , 3 .
c U I C j j C N < N r L \$ N a \$ 3 t . N 0 , j n I l w I a < 3 , a 0 C N I c a 3
j R @ q 3 j @ 3 3 u U 3 , j 3 0 C t N 3 \ h V 0 j w y n G C . S O O 4 W Y I c R
N 0 ; C j C G 8 R n N 0 j @ 3 R U j C L I I a < 3 , a 0 C N I t c n L
I z S : Y b U 3 , C ~ , I l w -

r @ w , a 0 C N I C N q a C N j c R B B

i @ 3 R a 3 L

H3†.v\$3 a3<nI a nN,RnNj \$I3 , a0CN I vY"n,t@ j@Cp
3\nC,RNcCcj3NjjR j@3 3uCcj3N,3 R8t sLCj@ta \$V3 , a0C

Ä M3sx7+a3cntj bRL3 3u LUI3c

Ä ` <@ q N N0 b@3I @ c@Rs30 j@ jÈ.8jPa3nCNB hrNÈCjL6
ÿ È @RI0c s@3a3 c CN j@3 ,RnNj \$I3 , c3. j@3a3 a
3uj3NcCRNc CN s@C,@ ŸNS0ÿ ICj@Bt0 a3cU3,jCq3lw
Ä i@3w I cR UaRq30 j@,jj.@3NÈ . ÈY `3,3NjIw.7Cc,@3
bRnGnU c@Rs30 V LRN< Rj@3acW j@ jj@3 c L3 ,RN
nN03a j@3 @wURjÈ3cCÈ,8

r @ w , a 0 C N I C N q a C N j c R B B

Ä `RCjL N c UaR \$lj3kGc s @3j @™a \$aCR Cc URccC\$13 jR U
- *ÆVcjCII RU3N6WY

Ä bR 8 a. b@3l @ < q3 j@3 \$3cj UUaRuCL jCRN jR N I
03q3IRU30 j@3 L3j@R0 R8 j3LUI j3 Cj3a jCRN 8Ra, C
j@3 CN3\n I p jw c c jCc~30. w3j CN @Cc LR08ld@3
\n3cjCRN j@ j Cc cjCII RU3N cGc C8 Cj Cc URccC\$
00CjCRN @ qC N<

Ä BN j@3 nN, RnNj \$13 CN , RNja cj. #l cc. ?wjjCN3N NO
CN7+8Ra nN, RnNj \$I È a B C j l n c UaR \$13L , N \$3 cRI
URcCjCq3. È Y 3 Y È C ß @ ð N È È Y

Ä 00CjCRN Ilw. 7Cc, @3a N0 bRnGnU @ q3 UaRq3 È j@ j
È Rj@3a a3l jCp3 È ß a N \$3 03, C030 jR @ È q Y q M I L 3 l w.
p j È p È È Y

r @ w , a 0 C N I C N q a C N j c R p j

Ä K R a 3 , a 0 C N I C N q a C N j c q C , R L \$ C N j R a B N j , @ 3 a R j 3
\$ 1 3 , c 3 j @ 3 8 R I I R s C N < @ R I 0 c -

i @ 3 R a 3 L V # c j R c y w « c G C W

H 3 j N 0 \$ 3 j s R 8 n N , j C R W c 3 C N w j @ N 0 a 3 q 3 N j n I l w 0 C 8 3 a
j @ 3 a 3 0 c , . c n , @ j @ j 8 R a I I V N 0 s a C j 3 W . j @ 3 N
N R N L C N] å] C P ħ , à n å ® - ^
, R q L C N] å] C P ħ , à n å - ^

i @ 3 N . C 8 s 3 0 3 ~ N 3 8 R a a \$ C j a a w t n N , R n N j \$ 1 3 a 3 < n
Ä N L È L C N]] C P , ě È à • n ® • - , ^
Ä , q È L C N]] C P , ě È à • n • - , ^

r @ w , a 0 C N I C N q a C N j c R p B C

i @ 3 8 R I I R s C N < @ R I 0 c -

T a R U R c C j C R N

Ä y È . N L È . N R N z Y

Ä , R q z . , q È . È Y

K R a 3 R q \$ a C C o j a R N < I w C N , , 3 c c C \$ I 3 . j @ 3 , R a a 3 c U R N
Ä + I n \$ q 3 a c C R N c -

Ä + n L L C N < c N 0 b @ 3 I @ 0 3 ~ N 3 0 j @ 3 , È n \$ N o j 3 È c N R I N 3 d R 8
< C q 9 N , n È s 3 c w j @ j , V , I n \$ 0 R L C M W 3 C 8 j @ 3 a 3 3 u
, I n \$ R N È c R j @ j . 8 R a j 3 n 3 d a w i , i N 0 0 3 ~ N j 3 0 È
N 0 , È , , R a 0 C N < I w Y i @ 3 w U a R q 3 0 -

r @ w , a 0 C N I C N q a C N j c R p B

i @ 3 R a 3 L

ÿ, È ÿ È , È . È N 0 È c c a 3 < n l a 0 N 0 , È È Y

Ä i @ 3 U c 3 n 0 R C N j 3 a c 3 , j C È C N N 0 B \$ 3 N 3 0 c j @ 3 L C N C L n L
8 L C l w R 8 c n \$ c 3 È s R ß @ ç @ R N < C N j 3 a c 3 , j C V C N B Y R B R 3 a
` f .] `] È Í ` C c n N \$ R n N 0 3 0 W N 0 N R U c 3 n 0 R
c C y È V C Y 3 Y M R n c 3 È > n , @ j @ f j - g . 8 R a g l n W Y

` < @ q N N 0 b @ 3 l @ U a R q 3 È l c R , j È j . ÿ È s @ 3 a 3
, È C c j @ 3 L C N C L n L c C y 3 R 8 8 L C l w R 8 , l n \$ c s C j @
R 8 c È Y ` 3 , 3 N j l w . C N E R C N j s R a G s C j @ 7 C c , @ 3 a N 0
j @ j j @ 3 a 3 C c L R 0 3 È s @ 3 a È Y

bRL3 a3cn

NRN t - ,R t - TM n t - TM n t - È
6 6
- t - TM t
6 6
t - - n t - - n t - 00i t - ,R q t
+C, @R « c / C <a L R N j @ 3 n È, Pa R N < \$ W B R a , 3 c c C \$ 13 Y

tAb ,Gc 8Ra

H3† \$3 cjaRN<lw CN ,,3ccC\$13Ÿ -aBIMQIRjCRONon\$Njfa33c
cn,@j@ "jn .à# n " fc#ICjjCN< N0j@3 ICLCjR8 cUIC
cUICjjCN<Y ICB f Y

Ä Bj @ c <RR0 8ncCRN UaRU3ajC3cY

Ä Bj @ cj@3 <3N3Ab I,Cyc3UaRU3ajvst@Ca33~N30\$w
l! CY3Y <Onq3EN NO bN .ĚAN L3 8Ra N 313Ě3N@G3N3
a3 . NOû C t B <Ě>AcI IR]ŪVk]. W cn,@j@
Â bk n û kRakII tY

i@3 Cj3a jC-RNCR@cnUURaj RB I3@ <ç@

Ä ;RR0 8ncCRN. cR., ta0GNS U@3c3aq30Y

Ä i@3 <3N3a ICy30 b ,Gc UaRU3ajw NÖM -a t,RdNsc3I\N3
cj@3 Rj@3a , a0CN lc CN j@3 3uj3N030YOC <a L a

NRN È - ,R8i È - TM n t - TM n t È
 6 6
 t - t - TM t
 6 6
 t - - n t - - n t - 00i È - ,Rqi È

7C < na3 e - 2 { 3 , j R 8 j @ È 7j , aG j C8RN , FC8N <

tAKCII3a7R

H3å \$3 tA,RLUI3j3 ~tj3a3R-NL3^É jR \$3 j@3 8RIIRsCN< 8Ra
+RN0CjC^ÉFNc cNA,IRc30 cn\$ARa3j3@3 c3j R8 CN,a3 cCN< c
t^É.cn,@ j@ j3q3aw NR03 , N \$å3Acj3N0j30NR NR03Y

IcR s3 s Njj@ j Cc IC\$^É nj^É. N0 8Ra a\$Cjalackw\$^É d<3
åAcUICj.cj@N\$åAcUICj^É CN

T a R U 3 a j C A K R I 8 3 a 8 F

Ä Bj @ c <RR0 8ncCRN \$ @ C , @3QLaUlc33a30Y
Ä ± È . s @ 3â c c j @ 3 , ln \$ ~lj3a. 00c +Rt@ 3N cn \$ c3j R8
Ä ± È <3N3aC, llw 00c N nN \$ RnN0b0È nW, jCRN Rq3a
Ä i @ 3 UaR n , jN ± È 00c tA+R @ 3N 8nN, jCRNY
Ä ± È @ c j @ 3 Una3 03, CcCRNûARUNBajjas-@ 3N Y È 3Y
N0^ Cc 8RaLnI CN j @ 3 8Ra, CN < I . N < s Cj3@ j @ 3 æ3
cj3L cn, @ j @ 3j, C0^cY3Â ^ Ra Â ® Y

i @ 3 < 3 N 3 a | C y 3 0 n l j a ~

H 3 † \$ 3 N n N , R n N j \$ I 3 , a 0 C N I Y

/ 3 ~ N C j C R N

^a t L C N] á] C C á c \$ c 3 8 R a n N C 8 R a t t ^ M j a ~ l j 3 a R N

m N C 8 R a L L 3 N c j @ j I l j @ 3 c 3 j c C N j @ Y 3 n t j R ô C l j 3 N @ l j a 3 - c l
R N . á f ô C c \$ c 3 C 8 û k a q ô N @ 3 a 3 a c á c n , @ j @ f j - û Y

i@3Ra3L V#aRRG3Ai wIRa. 7Cc, @3a. 7aC30L N.

bnUURtc c cnU3a,RLU ,j t-a0CNa\$.<nl a , a0CNtl-s.Cj@
N0 Cc , a0CN Ij@ j È jCα~3@3Nj@3a3 Cc 8Ra,CN< 3
s@C,@ , a0CN lc @ q3 NRj\$33N ,@ N<30 c jCc8wCN<-

t- a t — t ™ t - t “ t §, R q i È
00 i È NR N È , R 8 i È NO È

B8 CN 00Cj È RN j@3N s3 , N lcR UaRqC03 j@ j
¥ t © t * • t t-s@3 ä 3CctA,RLUI3j3 nIjatYIj3a RN

▶ :RjR -N

b 3 , j C R N I

i @ 3 C N 0 3 U 3 N 0 3 N , 3 N n L \$ 3 a

BN03U3N03Nj 8

/3~NCjCRN VMRj jCRNW

H3à \$3 8 LCIw R8 CN~NCj3 cn\$c3jc R8

Ä r3 03NR73 à j@3 8 LCIw R8 ~NCj3 U aj@ JBYN;GGRM
n77 à .à^f Í\ö^f b C ö n à ERL ^s@3ä^f b ö
C8 ö Nö^f b , ö Rj@3asCc3Y

Ä r3 a383àä^f C r77 à ^cj@3 8 LCIw R8 #RRI3 N ,RL
à ccR,C j30jR

/3~NCjCRN

8 LCàwf <, ÖCc , IICN03U3NC3NR a 8Ran3773 àw j@3 ä^fj
Cc CN~NCj3Y N CN03àÜCN03N0 8RLUCIw I CN03UC3N03Nj N
UaRU3alw ,RNj CN30 CN NRj@3a CN03U3N03Nj 8 LCIw

BN j@3 ~<na3. j@3 c3\j R8^a wC^Rn I c ú • N Ě í ... ‡, ú ž o

?Rs jR 00 N CN03U3N0

i@3 8RIIRsCN< a3cnljc a3 0n3 jR #a3N0I3-

H3LL

H3à \$3 N CN03U3N03Nj 8 LCIwY i@ 3N R@3a3j@cj @I3C 03
8RIIRsCN< UaRU3ajC3c-

sYé È \à^f C n7 à ^ &

IY7Ra 3q3anw<, >j@3a3 077 à cn, @ j@ j 3Cj @3Ra
à^f \$3IRN<é jR

r@3N3q3\$3 N CN03U3N03Nj 8 LCIwY 003 I c jCc8wCN< U
N0 VIW R8 j@3 I3LL \$Rq3é s3c @N@3U3N03N,3 0C <RM
C03 dcR,C j3àYR

i @ 3 8 R a ,

/3 ~ NCjCRN

H3à \$3 N CN03U3N03Nj 8 é C\$w N @NBq3U3N03N,3 0C <F
C03 I ccR,C j30 jR CjYé @ 3UNcc3jrc R8 !"l ú s @ 8 a3
" n <, > . ú n <é> Õ sCj@ 3uj3NcCRN a3I jCRN 03~N30 c 8
û . " ú8 NO R#Igv."ú8g ú NO #=" ÌËú âY

i@Cc URcésj CçA,3Nj3a30. cR Cj Ua3c3aq3c , a0CN IcY
8RIIR\$GNGIw 0C <RN ICy j-CRN UaRU3ajw

H3LL

H3ü \$3 i é <3N3aC,~lj3'aYG@3N C àû " û ɔdü^N
CN~NCj3 cη \$oc3,j@Rβ@ j<Cūà í \ ' _h ^Cc CN03U3N03Nj. s@C
n < , > à Ë . j @ 3 8 à ð i W ' _h ^Cc NRj CN03U3N03NjY
c , RaRll aw s3 R\$ j CN-

i@3Ra3L

V+?W Hβj v \$3 a3<nI a nN, RnNj \$I3 , a0CN lcY i@3a3 C
3uj3NcCRN ČN s™ C,t@ ~ v

<3N3aC, L uCL I CN03U3I

b@3I @ ,RNcján,j30 L uCL I CN03U3N03Nj8 LCIw. s
Ž *ÆCN LR03IR8_oY rCj@ 7Cc, @3a. s3 c@Rs30j@; †.?Rq3
8Ra 3u LUI3. @Cc ,RNcján,jCRN N jna Ilw <Cq3c aCc3
,IRc3_oA,, URć3†@C, @ <3N3aC, Ilw 0ERCNc L uCL I
s@C, @ jnaNc jR \$3 b ,Gc CN03cján,jC\$13Y

H3LL

H3à \$3 N CN03U3N03Nj 8 ã CIw jRN\$B Ij3@ 3 c3j R8 II 8nN,j
n 77 à 8Ra s@ Ę, @^f Cc ~NCj3. j@3N-

Coà \ f, C P n77 à à[¥]g à[^] Ę Cc ~NCj3
\ f, C ã Cc 03Nc37CàN ^

Cc N C03, IjRNs @ C, @ s3 a383a c j@3 CN03U3N03N, 3 03N
àY?3a3 s@3N s3 c w 037Nàc3s3N 3 N 03Nc3 a3cU3,jjR j@3
a3I jCRNY

i @ 3 < 3 N 3 a | C y 3

/3 ~ NCjCRN

H3à \$3 8 LCIw R8 nN\$RnN0\$R8nc\$Cy\$3-c R8

Ä r3 , I#7E à j@3 8 LCIw R8 8nNà jjRRsNCj@aRLL CN R18YcC

Ä ;Cq3Nn #7E à .à^f Í \ö^f b C ö n à ER L ^ s@3a3
ö^f b ö C8 ö Nö^f b t ö Rj@3asCc3Y

/3 ~ NCjCRN

8 LCàwf <t>cn, @ j@à]/ tCc , Ilt3AOC N03U3CN808 Np 8Ra 3q3
n #7E à . j@3 à3jCc nN\$RnN0\$R8nc\$Cy\$3-c R8
jR \$BAL uCL I CN03U3CN808 Np 8Ra 3q3
tACN03U3N03Nj 8 LCIwY

Finally...

In recent work with V. Fischer we have proved the following diagonalization lemma in order to decide the value of $\exists \text{fin}$ in the model where $\exists \text{fin}$ was small. Specifically, it is decided to be true . [▶ Go to model with small u](#)

Lemma

$\forall D \{ \exists D. \text{R_ADfD_AD}_{\{I.\}} \text{IS?} \exists D. \text{AT } M_{_}. \exists E \{ \text{E}_{_} \cdot \exists Dr. _A \exists D. \text{RMD_DrF} \cdot \exists Dr c, Dr. \exists M c \in _A \} \text{cADZ } j e \in \{ \{ c \exists D \{ \text{OD MD_DrF} \setminus \cdot \{ \text{OT } t \in _ = \{ \text{E}_{_} \cdot \text{AADA } \exists \{ \text{OT } \text{Icr} = _ M_c \{ \text{E}_{_} \} \{ \text{OD}_{_} \} \}$

1. $m \text{ o } \pi \text{ R_ADfD_AD}_{\{j\}}$
2. $\exists \text{ /MO } \text{fi } \pi t \in \text{Q} \{ \text{Q} \{ m \text{ o } \pi \text{ R_ADfD_AD}_{\{?\} \{ \text{OD}_{_} m \text{ t o } \pi _c j \}$

